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EXTENDED ABSTRACT

The study analyzed the expected value of a new firm project subject to a Markov-switching jump-diffusion stochastic. By extending the dynamic programming technique, the corresponding Hamilton-Jacobi-Bellman equation (HJB) of the problem turns out to be a system of partial integro-differential equations due to the extra terms arising from the Levy process and the Markov process. Analytical solutions for the value functions (expected value of the new firm project) which can be interpreted as a profit or cash flow rate that a firm earns continuously appear to be unavailable. Under the circumstances, numerical techniques have to be employed to find a closed form solution. The thrust of this work is the derivation of an explicit formula of the value function in terms of the so called *A*-hypergeometric series. Numerical examples are provided in the study based on data from several companies in The Bahamas.

Given the stock of a firm following a Jump-diffusion process with regime switching as follows, this study looks at the profit or cash flow rate that a firm earns continuously.

$$dS_t/S_t = \mu(z(t))dt + \sigma(z(t))dW_t + F(u)dq_t$$

This amount is especially important in economic analyses as the problem can be interpreted as determining the optimal exercise date *T* of an American call option with the infinite expiration to maximize the profit function. These problems are relevant both from a practical point of view and from the mathematical challenge they present.

The motivation of this work is due to the following: Many well-documented analyses have shown that the stock market periodically oscillates between the “bull market” and the “bear market”. It is therefore conceivable that the drift and the volatility in a bull market takes different values from those in a bear market and that the stock can make a jump due to the transaction. The existing approach, as highlighted by Vollert (2003), does not consider the “bull market” and the “bear market”. It would seem then that the jump diffusion with regime switching is a better model.

In this study, the expected value of the new firm project was evaluated

$$\phi(s, x, k) \;=\; \mathbb{E}\left[\int_s^{\tau_k} f(t, U(t), z(t)) \mid U(s) = x, z(s) = k\right]$$

by using dynamic programming (Bellman, 1952) with finite random horizon and state variable dynamics given by a Markov-switching jump-diffusion as follows:

$$dU_t = \mu(z(t))dt + \sigma(z(t))dB_t + J(u)dN_t \text{ with } U_t = \ln(S_t)$$

The researchers found the associated Hamilton-Jacobi-Bellman (HJB) equation to be a system of partial integro-differential equation

$$\frac{1}{2}\sigma^2(k)\frac{\partial^2\phi(s,x,k)}{\partial x^2} + \frac{\partial\phi(s,x,k)}{\partial s} + \mu(k)\frac{\partial\phi(s,x,k)}{\partial x} + \sum_{l=1}^{m}\phi(s,x,l)$$
$$+\lambda\int_{\mathbb{R}}\phi(s,x+u,k) - \phi(s,x,k)F(u)du + f(s,x,k) = 0$$

with the boundaries condition as follows:

$$\phi(s,0,k) = a, \quad \phi(0,x,k) = b, \quad \frac{\partial\phi(s,0,k)}{\partial x} = c, \quad \frac{\partial^2\phi(s,0,k)}{\partial x^2} = d.$$

The main contribution of the study is the solution of the system (HJB) in terms of *A*-hypergeometric series (Sturmfels, 2000).

In conclusion, this paper introduced the challenge of uncontrolled jump-diffusion process with regime switching which is a problem of probability of first hitting time. The study found the explicit formula of the profit function by combining some method in analysis and combinatorics. The jump-diffusion process with regime switching seems to be very appropriate and useful for representation of market movements between various states (e.g., growth, crisis). For the practical implementation of such models it would be interesting to use Hamilton's (1988) method which is slightly different to the usual linear *AR*(m) to estimate the parameters. Further, the study concluded with an application to a problem in some companies in The Bahamas.

Keywords: *Hamilton-Jacobi-Bellman equation, Jump-diffusion, regime switching, stock analysis*

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References

Bellman, R. (1952). On the theory of dynamic programming. *Proceedings of the National Academy of Sciences*, *38*(8), 716-719. https://doi.org/10.1073/pnas.38.8.716

Hamilton, J. D. (1988). Rational-expectations econometric analysis of changes in regime: An investigation of the term structure of interest rates. *Journal of Economic Dynamics and Control*, *12*(2-3), 385-423. https://doi.org/10.1016/0165-1889(88)90047-4

Sturmfels, B. (2000). Solving algebraic equations in terms of A-hypergeometric series. *Discrete Mathematics*, *210*(1-3), 171-181. https://doi.org/10.1016/S0012-365X(99)00126-0

Vollert, A. (2003). *A stochastic control framework for real options in strategic valuation*. Boston: Birkhauser.

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Dr. Moussa Kounta joined University of The Bahamas as an assistant professor in the Department of Mathematics in August 2014. He holds a Bachelor's degree (2004), a Master's degree (2005) in mathematical physics and a Diplôme d'études approfondies DEA (2007) in commutative algebra with honours, all from the University of Dakar, Senegal. After some preliminary work in max-plus algebra in 2007, he was invited to join a Ph.D. programme in the area of max-plus algebra under the supervision of Professor Edouard Wagneur at the Polytechnique Montréal in Canada. He completed his Ph.D. in Mathematics at the Université de Montréal in 2012 under the supervision of Professor Mario Lefebvre (Polytechnique Montréal). During his Ph.D. programme he published several research papers in high impact international mathematical journals. After his Ph.D., he joined as a postdoctoral research fellow in Professor François Watier's group (Équipe de modélisation stochastique appliquée (EMoStA) at the Université de Québec in 2013. He has broad research interests in diverse mathematical fields including max-plus algebra, stochastic process, stochastic optimization and probability. He is currently working in stochastic process and its application in finance, hydrology, etc. in collaboration with his former Ph.D. and postdoctoral supervisors. He supervised two undergraduate students at University of The Bahamas in stochastic process applied in finance (interest rate of the Central Bank of The Bahamas) and statistics (forecasting the real GDP of The Bahamas).

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