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# REAL OPTIONS OF A STOCK FOLLOWING A JUMP DIFFUSION WITH REGIME SWITCHING

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## EXTENDED ABSTRACT

The study analyzed the expected value of a new firm project subject to a Markov-switching jump-diffusion stochastic. By extending the dynamic programming technique, the corresponding Hamilton-Jacobi-Bellman equation (HJB) of the problem turns out to be a system of partial integro-differential equations due to the extra terms arising from the Levy process and the Markov process. Analytical solutions for the value functions (expected value of the new firm project) which can be interpreted as a profit or cash flow rate that a firm earns continuously appear to be unavailable. Under the circumstances, numerical techniques have to be employed to find a closed form solution. The thrust of this work is the derivation of an explicit formula of the value function in terms of the so called  $A$ -hypergeometric series. Numerical examples are provided in the study based on data from several companies in The Bahamas.

Given the stock of a firm following a Jump-diffusion process with regime switching as follows, this study looks at the profit or cash flow rate that a firm earns continuously.

$$dS_t/S_t = \mu(z(t))dt + \sigma(z(t))dW_t + F(u)dq_t$$

This amount is especially important in economic analyses as the problem can be interpreted as determining the optimal exercise date  $T$  of an American call option with the infinite expiration to maximize the profit function. These problems are relevant both from a practical point of view and from the mathematical challenge they present.

The motivation of this work is due to the following: Many well-documented analyses have shown that the stock market periodically oscillates between the “bull market” and the “bear market”. It is therefore conceivable that the drift and the volatility in a bull market takes different values from those in a bear market and that the stock can make a jump due to the transaction. The existing approach, as highlighted by Vollert (2003), does not consider the “bull market” and the “bear market”. It would seem then that the jump diffusion with regime switching is a better model.

In this study, the expected value of the new firm project was evaluated

$$\phi(s, x, k) = \mathbb{E} \left[ \int_s^{T_k} f(t, U(t), z(t)) \mid U(s) = x, z(s) = k \right]$$

by using dynamic programming (Bellman, 1952) with finite random horizon and state variable dynamics given by a Markov-switching jump-diffusion as follows:

$$dU_t = \mu(z(t))dt + \sigma(z(t))dB_t + J(u)dN_t \text{ with } U_t = \ln(S_t)$$

The researchers found the associated Hamilton-Jacobi-Bellman (HJB) equation to be a system of partial integro-differential equation

$$\begin{aligned} \frac{1}{2}\sigma^2(k)\frac{\partial^2\phi(s,x,k)}{\partial x^2} + \frac{\partial\phi(s,x,k)}{\partial s} + \mu(k)\frac{\partial\phi(s,x,k)}{\partial x} + \sum_{l=1}^m \phi(s,x,l) \\ + \lambda \int_{\mathbb{R}} \phi(s,x+u,k) - \phi(s,x,k)F(u)du + f(s,x,k) = 0 \end{aligned}$$

with the boundaries condition as follows:

$$\phi(s,0,k) = a, \quad \phi(0,x,k) = b, \quad \frac{\partial\phi(s,0,k)}{\partial x} = c, \quad \frac{\partial^2\phi(s,0,k)}{\partial x^2} = d.$$

The main contribution of the study is the solution of the system (HJB) in terms of  $A$ -hypergeometric series (Sturmfels, 2000) defined as follows:

$$\begin{aligned} X_{j,\xi} &= \xi[a_{i_{j-1}}^{1/d_j} a_{i_j}^{-1/d_j}] + \frac{1}{d_j} \sum_{k=2}^{d_j} \xi^k [a_{i_{j-1}+k-1}^{(k-d_j)/d_j} a_{i_j}^{-k/d_j}] + \frac{1}{d_j} \left[ \frac{a_{i_{j-1}}}{a_{i_j}} \right] \\ |a_{i_{j-1}}|^{i_j-k} |a_{i_j}|^{k-i_{j-1}} &\leq M |a_k|^{d_j} \quad \text{for all } a \leq j \leq r \text{ and } k \notin \{i_{j-1}, i_j\}. \\ [a_0^{u_0} a_1^{u_1} \dots a_n^{u_n}] &= \sum_{(v_0, \dots, v_n) \in \mathcal{L}} \prod_{i=0}^n a_i^{u_i+v_i} \Gamma(u_i+1)/\Gamma(u_i+v_i+1) \end{aligned}$$

Where  $L$  denotes the integer kernel of  $A = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$ .

In conclusion, this paper introduced the challenge of uncontrolled jump-diffusion process with regime switching which is a problem of probability of first hitting time. The study found the explicit formula of the profit function by combining some method in analysis and combinatorics. The jump-diffusion process with regime switching seems to be very appropriate and useful for representation of market movements between various states (e.g., growth, crisis). For the practical implementation of such models it would be interesting to use Hamilton's (1988) method which is slightly different to the usual linear  $AR(m)$  to estimate the parameters. Further, the study concluded with an application to a problem in some companies in The Bahamas.

**Keywords:** *Hamilton-Jacobi-Bellman equation, Jump-diffusion, regime switching, stock analysis*

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## References

- Bellman, R. (1952). On the theory of dynamic programming. *Proceedings of the National Academy of Sciences*, 38(8), 716-719.  
<https://doi.org/10.1073/pnas.38.8.716>
- Hamilton, J. D. (1988). Rational-expectations econometric analysis of changes in regime: An investigation of the term structure of interest rates. *Journal of Economic Dynamics and Control*, 12(2-3), 385-423.  
[https://doi.org/10.1016/0165-1889\(88\)90047-4](https://doi.org/10.1016/0165-1889(88)90047-4)
- Sturmfels, B. (2000). Solving algebraic equations in terms of A-hypergeometric series. *Discrete Mathematics*, 210(1-3), 171-181.  
[https://doi.org/10.1016/S0012-365X\(99\)00126-0](https://doi.org/10.1016/S0012-365X(99)00126-0)
- Vollert, A. (2003). *A stochastic control framework for real options in strategic valuation*. Boston: Birkhauser.

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